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Predictive Distribution of Adjustment for Life Expectancy Change

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ABSTRACT

An increase in life expectancy can put a strain on the finances of a defined-benefit pension system. A possible way to alleviate the effect is to adjust the levels of pensions to changes in life expectancy. However, as future development of life expectancy cannot be predicted without error, we can only assess the possible risks probabilistically. A predictive distribution for adjustment factors is derived in the Finnish setting. The distribution indicates that by year 2050 we can expect an adjustment factor of 0.87. The width of an 80 % prediction interval is 20 percentage points, if past volatility of mortality is used as a guide.

1 INTRODUCTION

An increase in life expectancy can put a strain on the finances of a defined-benefit pension system. In a pay-as-you-go system this may mean increasing the contribution levels of the current workers. In a funded system, the development of life expectancy must be anticipated in setting the premiums of the future retirees. In Finland, the earnings related pension system is partially pre-funded, so both aspects are relevant.

In anticipation of future gains in life expectancy, a law has been passed in Finland that automatically adjusts pensions, if life expectancy changes. The aim of the legislation is to preserve the net present value of future pensions. As the development of future mortality is uncertain, it is of interest to consider the level of uncertainty one can expect in future life expectancy and in net present values. The main purpose of this paper is to provide a stochastic analysis of the proposed adjustment factors in the context of the current Finnish proposals.

2 NET PRESENT VALUE OF PENSION

Let $a > 0$ be a base age for pension. In our application, we will have $a = 62$ (in exact years). For any $x \geq 0$, define $p(x)$ as the conditional probability of surviving to age $a + x$ given survival to a . It follows that $p(0) = 1$, and $p(x)$ is monotone decreasing. Then, life expectancy at age a equals

$$\int_0^{\infty} p(x) dx.$$

Let r be a discount rate such that the value of a euro received in age $a + x$ is worth e^{-rx} at age a . In our application we will have $r = 0.02$. Suppose a pension is paid continuously, at the rate of one euro per year. Then, the expected value of the whole pension is

$$\int_0^{\infty} p(x) e^{-rx} dx.$$

This is also called the *net present value* of the pension.

In practical calculations, the integral defining the net present value has to be approximated in some way. Suppose estimates of $p(x)$ for $x = 1, 2, \dots$ are available. The simplest approach approximates the integrand by a straight line in intervals $[0, 1]$, $[1, 2]$, etc. This is the so-called trapezoidal method. The approximate value is

$$x = 1/2 + \sum_{x=1}^{\infty} p(x) e^{-rx}.$$

In practice, the values $p(1), p(2), \dots$ must be estimated based on mortality data. Let x be an integer, and define m_x as the age-specific mortality rate in age $[x, x + 1)$. Then, the so-called actuarial estimator for surviving from exact age x to exact age $x + 1$ is $(2 - m_x)/(2 + m_x)$. It follows that we can estimate

$$\hat{p}(x) = \prod_{y=0}^{x-1} \frac{2 - m_y}{2 + m_y}.$$

Thus, an estimator of the net present value is

$$\hat{x} = 1/2 + \sum_{x=1}^{\infty} \hat{p}(x) e^{-rx}.$$

We note that the actuarial estimator is actually based on a hypothesis that $p(\cdot)$ is linear in the interval $[x, x + 1)$. Strictly speaking, this contradicts the linearity hypothesis leading to the formula for $?$. A more complex formulation for $?$ is given in Appendix I that avoids the apparent contradiction. Numerically the difference is unimportant, however.

3 A PERIOD ESTIMATE OF NET PRESENT VALUE

In practice, estimates of net present values are computed based on past data. Consider the cohort of individuals who become a years old during a calendar year t . In our application the years of interest are $t \geq 2009$. To calculate the net present value for year t , mortality data from the 5-year period $[t - 6, t - 2]$ is used.

Let $D_x(u)$ be the number of deaths in age x during year u , and let $K_x(u)$ be the number of person years lived in the population, in age x , during year u . The age-specific mortality in age x during $[t - 6, t - 2]$ can be estimated as

$$\hat{m}_x(t) = \sum_{u=t-6}^{t-2} D_x(u) / \sum_{u=t-6}^{t-2} K_x(u).$$

Using this estimate, one can estimate the probabilities of survival relevant for year t as

$$\hat{p}(x, t) = \prod_{y=0}^{x-1} \frac{2 - \hat{m}_y(t)}{2 + \hat{m}_y(t)}.$$

Then, the net present value for year t can be estimated as

$$\hat{x}(t) = 1/2 + \sum_{x=1}^{\infty} \hat{p}(x, t) e^{-rx}.$$

We remark that an alternative method of estimating age-specific mortality would be to average the annual rates $D_x(u)/K_x(u)$. We show in Appendix II that the latter method is more susceptible to random variation, if the person years change from year to year. The effect is small, however, unless the relative sizes of the age-groups are very different.

4 ADJUSTMENT FACTOR FOR MORTALITY CHANGES

If mortality changes, then the net present values may also change. One method to maintain a fixed level is to adjust the rate at which pension is paid. Let T denote the base year. In our case, we will have $T = 2009$. Then, if pensions are multiplied by a factor $A(t) = \mathbf{x}(T) / \mathbf{x}(t)$ for $t \geq T$, their net present values do not change.

Since the development of future mortality is uncertain, the values of $A(t)$ cannot be known accurately at present time. However, a probabilistic description of how they are likely to behave, can be given. We will discuss two issues. First, since mortality rates are subject to random variation, in Section 5 we will determine the level of year to year variation due to this source. Second, a more important source of uncertainty is caused by unexpected changes in the trends of mortality. The models and assumptions underlying such an analysis will be given in Section 6 with results in Section 7.

5 SAMPLING VARIATION IN ADJUSTMENT FACTORS

Even if the underlying forces of mortality were constant over time, the number of deaths would vary from year to year due to random variation. This type of sampling variation is usually modeled using the Poisson distribution. We will determine, how much should one expect the adjustment factors to vary from year to year, if this were the only source of variation.

As before, let $D_x(u)$ be the number of deaths in age x during year u , and let $K_x(u)$ be the number of person years lived in the population, in age x , during year u . Under the Poisson model of deaths we assume that $D_x(u) \sim \text{Po}(m_x(u) K_x(u))$, where $m_x(u)$ is the age-specific mortality rate in age x , for year u . The deaths are assumed to be independent over age and time. For the purpose of the following analysis, let us assume that neither the rates nor the person years change from year to year, so that we can write $D_x(u) \sim \text{Po}(m_x K_x)$. In this case, we have the estimate

$$\hat{m}_x(t) = \sum_{u=t-6}^{t-2} D_x(u) / 5 K_x,$$

where the $D_x(u)$'s are independent. Let us consider $\hat{m}_x(T)$ as being fixed. Using simulation, we can now study the induced variation for $t > T + 6$, when the data involved in the determination of $\hat{m}_x(T)$ is no more part of the calculation. For years $T \leq t \leq T + 6$ the induced variation is less.

To define the setting, we will use female lifetable estimated for 2001 to determine the age-specific mortality rates in each age. The rates were available for ages 62-99. For higher ages, an average rate was used that yields the remaining life expectancy of 1.9 years in exact age 100. The error caused by the last mentioned approximation is small, since the probability of surviving from age 62 to age 100 is only 0.013. The resulting remaining life expectancy in age 62 is 22.24 years. The net present value of a unit pension is

17.50 euros, when the discount rate is 0.02. In the following calculation we will take $\varphi(T) = 17.50$. This value close to what we expect for the combined male and female data in 2025.

The age-structure of the population was also determined by the female life table. The size of the population was fixed at 1,000,000, a close approximation of the size of the age-group 62+ (both sexes combined) in Finland in 2002. The expected number of deaths in this population is 43,100. A Poisson count with this expectation has a coefficient of variation of 0.005. Or, with a probability of 95 % the annual deaths are within $\pm 1\%$ of the expected value.

Under unchanging mortality we expect approximately that $A(t) = \varphi(T)/\varphi(t) \approx 1$ for all t . Actually, since $A(t)$ is a convex function of the random denominator $\varphi(t)$, we expect by Jensen's inequality that $A(t) \geq 1$. Define the change in the adjustment factor for two calendar years that are $k = 1, 2, \dots$ years apart, as $\Delta(k) = A(t+k) - A(t)$. We expect that $\varphi(k) \approx 0$ for all k , but it is of interest to determine how much variation there might be from year to year.

Based on 1,000 simulations, the results are as follows. First, the standard deviation of $A(t)$ is 0.007. Second, the correlations $\text{Corr}(A(t+k), A(t))$ are 0.80, 0.60, 0.43, 0.22 for $k = 1, 2, 3, 4$. For $k \geq 5$ the theoretical correlation is zero. Third, the standard deviations of the $\varphi(k)$'s are 0.00042, 0.00060, 0.00072, 0.00084, 0.00097 for $k = 1, \dots, 5$.

The interpretation is that for adjustment factors that are sufficiently far apart to be independent (or, for $k \geq 5$), pure sampling variability induces a standard deviation of the difference of approximately 0.1 %. Due to the use of 5-year data periods, adjustment factors that are closer ($k < 5$) are autocorrelated, so their differences are smaller. In particular, pure sampling variability induces a standard deviation of approximately 0.04 % between consecutive years. The distributions are approximately normal, so with a probability of 95 %, pure sampling variability induces a difference that does not exceed $\pm 0.08\%$ into annual adjustment factors.

In summary, Poisson type sampling variability is small. If single year data were used instead of using data from five consecutive years, then all standard deviations would have to multiplied by approximately $5^{1/2} = 2.24$. Hence, the use of a five year period has the dual effect of both reducing variability and smoothing the correction factors over time.

6 MODELS, ESTIMATES, AND ALTERNATIVE ASSUMPTIONS

6.1 A Model for the Predictive Distribution

The essential source of uncertainty concerning the future values of the correction factors comes from the fact that the future mortality rates $m_x(t)$ are unknown. Statistically, we can view them as random variables, and derive a predictive distribution that reflects our uncertainty concerning their forecasted values. Given the low level of Poisson type sampling variability, this uncertainty is also subsumed under the randomness of the rates. This simplifies the probabilistic structure by removing one layer of variability. One way for specifying random rates was developed in Alho and Spencer (1997). It was applied to Finnish data in Alho (1998), and updated in Alho (2002). The calculations to be presented here will rely on the updated estimates.

Suppose a forecast is made at time $t_0 < T$. In our case, t_0 will be the beginning of the year 2002. We will assume that for $t > t_0$ the random rates are of the form $m_x(t) = \hat{m}_x(t) \exp(Y_x(u))$, where $\hat{m}_x(t)$ is the point forecast of age-specific mortality $u = t - t_0$ years ahead, and $Y_x(u)$ represents its relative error. The point forecast was specified by assuming that the rate of decline observed for each age and sex during recent past would continue indefinitely. The empirical rates of decline were smoothed using the procedure RSMOOTH of Minitab, before calculating the point forecast. This method is similar in spirit - although not in technical details - to the approach suggested by Lee and Carter (1992).

The uncertainty was specified as follows. First, represent relative error in terms of error increments, $Y_x(u) = e_x(1) + \dots + e_x(u)$.

Then, assume that the increments are of the form $e_x(u) = S_x(u)(h_x + d_x(u))$, where $S_x(u) > 0$ is a scale parameter that determines the standard deviation of the error increment. Fixing x , the random components h_x and $d_x(u)$, $u = 1, 2, \dots$

are all assumed to be independent and normally distributed with $\epsilon_x \sim N(0, \sigma_x)$ and $d_x(u) \sim N(0, 1 - \sigma_x)$, where $0 < \sigma_x < 1$. As functions of x , the components ϵ_x , and the components $d_x(u)$ follow autoregressive processes of order 1 (or AR(1) process).

To gain intuitive understanding of the model, suppose that the scales would not depend on u , or $S_x(u) = S_x$. Then, we can write the relative error in the form $Y_x(u) = S_x(\mathbf{h}_x u + (\mathbf{d}_x(1) + \dots + \mathbf{d}_x(u)))$.

In other words, we have a sum of a line with random slope and a random walk. The relative weight given to the random line part and the random walk part is determined by σ_x and the overall level of error is determined by S_x . We have that $\text{Var}(Y_x(u)) = S_x^2(\sigma_x u^2 + (1 - \sigma_x)u)$, a second order polynomial.

6.2 Empirical Estimates

In the practical application of the model the scales were not assumed to be constant, but they were assumed to be the same for all ages and both sexes. They were estimated from Finnish data from 1900-1994 to match the level of error in trend extrapolation forecasts in the past (cf., Alho 1998). The relative weights of the random components were estimated as corresponding to $\sigma_x = 0.149$ for all ages and both sexes, and to a good approximation the standard deviation of the relative error, or coefficient of variation, was $0.032(0.15u^2 + 0.85u)^{1/2}$. At $u = 30$, this implies a coefficient of variation of 0.40, for example. This indicates a high level of uncertainty in mortality forecasting but, as we shall see below, life expectancy is much less volatile.

The autocorrelation parameters for the ϵ -terms were 0.945 for males and 0.888 for females. The autocorrelation parameters for the d -terms were 0.977 for males and 0.979 for females. For a fixed x , the cross-correlation between the ϵ -terms of the males and females were 0.795. Similarly, for fixed x and u , the $d_x(u)$ -terms had the correlation of 0.795 across sexes.

In the year 2000 life expectancy for females was 81.0 and for males 74.1 years. Using the parameters estimated from the past data we can calculate a predictive distribution of life expectancy via simulation. Based on 3,000 simulation rounds the median (Md), the first and third quartiles (Q_1 , Q_3), and the first and ninth deciles (d_1 , d_9), for the years 2030 and 2050 are as follows (Alho 2002):

sex	year	d_1	Q_1	Md	Q_3	d_9
female	2030	82.7	83.8	85.0	86.2	87.3
	2050	83.3	84.6	86.7	88.4	90.1
male	2030	75.9	77.5	79.3	81.0	82.6
	2050	76.7	79.0	81.8	84.3	86.4

The interpretation is that there is a 10% chance that female life expectancy will exceed 90.1 in 2050, for example. For comparison, a recent U.N. forecast for Finland assumes 86.1 and 79.8 for females and males, respectively, in 2045-2050 (United Nations (2001), p. 633). Given the length of the forecast period, the difference between the U.N. forecast and ours must be considered negligible.

6.3 Likelihood of Faster Decline in Mortality

Recently, Vaupel and Oeppen (2002) have presented evidence of the development of the so-called “best practice life expectancy”. This is the life expectancy of the country that at any given time has the longest life expectancy. Vaupel and Oeppen show that for females the curve goes almost linearly from the value of 45 years observed in Sweden in 1840, to 85 years observed in Japan in 2000. Or, life expectancy has improved by approximately 0.25 years annually.

In Finland, on the other hand, in 1930-2000, female life expectancy increased by $81 - 57 = 24$ years. This is approximately 0.34 years annually over the 70 year period, or more than the improvement in the best practice countries. However, when we look at the first 40 years of the period, the increase is $75 - 57 = 18$ years, or 0.45 years annually. During the latter 30 years the increase was $81 - 75 = 6$ years, or 0.20 years annually. The forecast of Alho (2002) assumed that age-specific mortality rates continue to decline at the rate they have declined during the past 15 years. During this period female life expectancy improved by only $81.0 - 78.6 = 2.4$ years, or 0.16 years annually. This rate would imply an improvement to 88 years by 2050. This is at 0.70 fractile of the predictive distribution.

The evidence presented by Vaupel and Oeppen is intriguing. Based on our analysis of the Finnish data, a resumption of an increase in life expectancy at the rate of 0.25 years annually would require a major acceleration in the decline of old-age mortality. Such an acceleration is not evident now and our analysis shows it would be unusual in historical Finnish perspective. Yet, we will indicate below, how our results can be adjusted, if more credence is put on such major improvements.

7 VARIATION DUE TO RANDOM RATES

7.1 Simulation Set-up

In Section 6.1 we defined the form of the predictive distribution, and in Section 6.2 presented empirical estimates for the model parameters. Adjustment for changes in life expectancy is made for both sexes using the same net present values. They are computed from age-specific data that are not disaggregated by sex.

To simplify the numerical analysis we will use two approximations. First, we will assume that the cross-correlation of error increments between males and females is = 1, instead of the empirically estimated value of 0.795. As discussed in Appendix III, this will not inflate standard deviations by more than 5 %. The assumption implies that the relative errors of the males and females can be taken to be the same. Therefore, sex only needs to be considered when computing the point forecast for the average age-specific mortality of the two sexes. The required sex ratios in ages $x \geq a$ are available from the sex ratios of the median forecasts. We denote the resulting values by $\hat{m}_x(t)$, and the resulting model for the age-specific rate during $t > t_0$ is

$$\tilde{m}_x(t) = \hat{m}_x(t) \exp\left(h_x \sum_{j=1}^{t-t_0} s_x(j) + \sum_{j=1}^{t-t_0} d_x(j) s_x(j)\right).$$

Second, define $\hat{K}_x(u)$ as the median forecast of the person years in age x during year u . We simplify the calculations by using these values (instead of the random population values that are not currently available from the output of PEP) to weight the random rates in the calculation of net present values.

After the approximations, the average mortality rate needed in the calculation of the net present values for years $t \geq T$ is simply

$$\tilde{m}_x(t) = \sum_{u=t-6}^{t-2} \tilde{m}_x(u) \hat{K}_x(u) / \sum_{u=t-6}^{t-2} \hat{K}_x(u).$$

These are random variables in the current setting. They allow us to compute probabilities of survival via

$$\tilde{p}(x, t) = \prod_{y=0}^{x-1} \frac{2 - \tilde{m}_y(t)}{2 + \tilde{m}_y(t)}.$$

net present values via

$$\tilde{x}(t) = 1/2 + \sum_{x=1}^{\infty} \tilde{p}(x, t)e^{-rx},$$

and adjustment factors for $t > T$ via

$$\tilde{A}(t) = \tilde{x}(T) / \tilde{x}(t).$$

7.2 Results

The practical calculations are carried via stochastic simulation using the program Minitab. The median, the first and third quartiles, and the first and ninth deciles, for the predictive distribution of the adjustment factors in 2010-2050 are as follows:

year	d ₁	Q ₁	Md	Q ₃	d ₉
2010	0.990	0.992	0.995	0.998	1.001
2020	0.915	0.933	0.953	0.973	0.995
2030	0.863	0.884	0.918	0.951	0.985
2040	0.814	0.842	0.889	0.936	0.983
2050	0.778	0.811	0.865	0.921	0.982
2060	0.751	0.787	0.843	0.905	0.974

Figure 1 has the corresponding data for the years 2010-2060. We expect the adjustment factor to decline to about 0.87 in 2050, with an 80% prediction interval [0.78, 0.98]. These intervals are valid provided that the volatility of the trends of mortality during the next 50 years does not exceed the volatility of mortality during 1900-1994.

Recall that the 0.90 fractile for life expectancy at birth in 2050 is 90.1 for females. This would imply an annual increase of $9.1/50 = 0.18$ years. An optimist in mortality reduction who believes in a reversal of the recent slowdown of mortality reduction may use the first decile (0.78) as a benchmark to consider how to adjust the predictive distribution to better match his or her beliefs.

In Section 3 we showed that pure Poisson variation would produce a standard deviation of only 0.04 % between the adjustment factors of consecutive years. A direct comparison to the random rates case can be made by calculating the standard deviation of the adjustment factor for year 2010. It has standard deviation of 0.0043 or it is 0.43 %. In other words, the effect of the uncertainty in the rate clearly dominates the Poisson variability.

8 DISCUSSION

We have provided here what appears to be the first stochastic analysis of a life expectancy adjustment that has recently been passed by the parliament in Finland. A similar adjustment has earlier been enacted in Sweden. Our calculations are based on earlier work on the difficulty of forecasting age-specific mortality using simple trend extrapolation methods. Our main result is a predictive distribution for the adjustment factor that reflects the past uncertainty of such trend forecasts, with little or no subjective input. Judgmental adjustments can certainly be introduced to modify the distribution, but then the empirical character of the result is materially altered.

For each new retirement cohort the adjustment factor is calculated using by the most recent mortality data available. This is period calculation, i.e., no attempt at forecasting the eventual life expectancy of the cohort is made. This can be justified on practical grounds, since cohort calculations would require forecasts going approximately 40 years into the future. Such forecasts are necessarily uncertain, and may lead to disagreements among the various parties involved.

Yet, despite such uncertainties, the pension monies will be paid to actual cohorts. An intriguing problem for future work would be to assess the predictive distribution of the difference between net present values as calculated based on cohort experience, and net present values as calculated based on the most recent data.

Predictive distributions can be valuable in two types of applications. First, if they can be communicated to the working age population, they may help workers to prepare better for their own retirement. Second, predictive distributions are needed for the economic analyses of retirement decisions, so that the risk aversion of the future retirees can properly be accounted for.

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APPENDICES

I. Assume that $p(x + y) = p_x + (p_{x+1} - p_x)y$ for $y \in [0, 1)$, so $p(\cdot)$ is linear on $[x, x + 1)$ with $p(x) = p_x$ and $p(x + 1) = p_{x+1}$. By a direct calculation one can show that

$$\int_x^{x+1} p(z) e^{-rz} dz = p_x e^{-rx} (1 - e^{-r}) / r + (p_{x+1} - p_x) (1 - r e^{-r} - e^{-r}) / r^2.$$

This leads to an alternative to ?. In the case $r = 0.02$, the formula gets the form

$$x' = 0.496683 + 1.00003 \sum_{x=1}^{\infty} \hat{p}(x) e^{-0.02x}.$$

II. Assume that $D(u) \sim \text{Po}(mK(u))$ are independent, and define

$$\hat{m} = \sum_{u=1}^n D(u) / \sum_{u=1}^n K(u), \quad \tilde{m} = \frac{1}{n} \sum_{u=1}^n D(u) / K(u).$$

Therefore,

$$\text{Var}(\hat{m}) = m / \sum_{u=1}^n K(u), \quad \text{Var}(\tilde{m}) = \frac{m}{n^2} \sum_{u=1}^n 1 / K(u).$$

The first variance is smaller because by Jensen's inequality, we have that

$$1 / \frac{1}{n} \sum_{u=1}^n K(u) \leq \frac{1}{n} \sum_{u=1}^n 1 / K(u), \quad \text{with equality only if } K(1) = \dots = K(n).$$

III. Consider two random variables with $\text{Var}(X_1) = 1$ and $\text{Var}(X_2) = c^2$, where $0 < c \leq 1$. Assume that $\text{Corr}(X_1, X_2) = \rho$. Then, $\text{Var}(X_1 + X_2) = 1 + c^2 + 2\rho c$. If we would have $\rho = 1$, the variance of the sum would be $(1 + c)^2$. Define the ratio of variances as $f(c) = (1 + c^2 + 2\rho c) / (1 + c)^2$. We find that $f'(c) = 0$, only if $c = 1$. The sign of the derivative changes from negative to positive, so this is the minimum. The minimum value is $f(1) = (1 + \rho) / 2$. For $\rho = 0.795$ this is 0.897, so the corresponding ratio of the standard deviations is 0.947. Hence, assuming a

perfect correlation does not inflate the variance by more than 5 %, no matter what the ratio of the variances is.

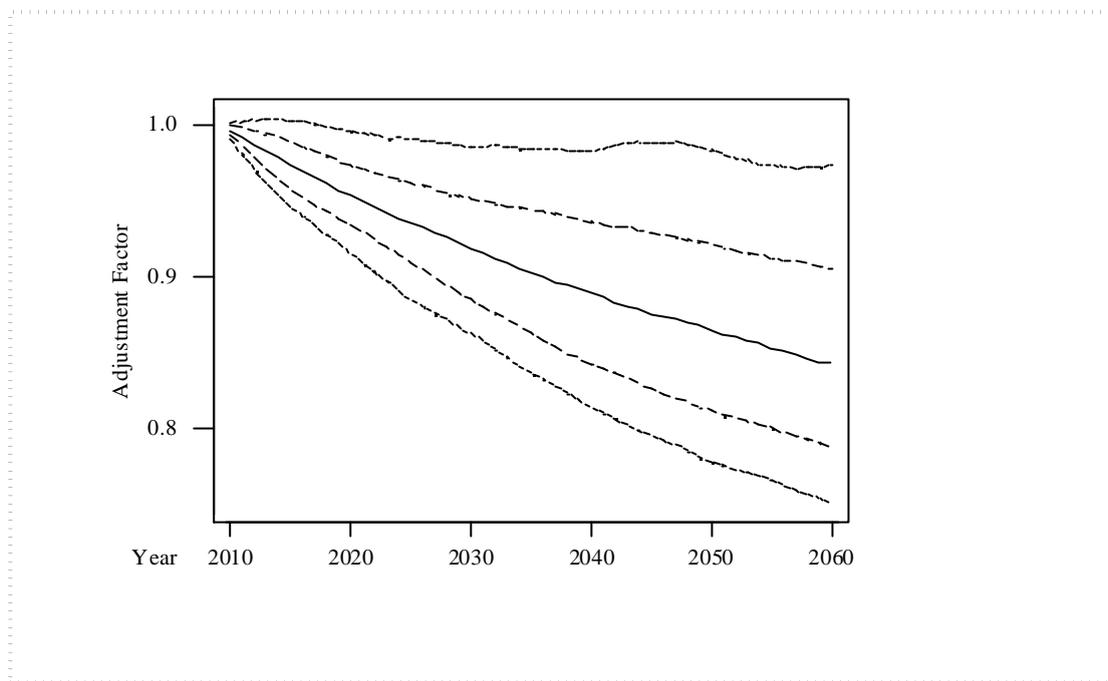


Figure 1. Predictive Distribution of the Adjustment Factor in 2010-2060: Median (Solid), First and Third Quartiles (Dashed), and First and Ninth Deciles (Dotted).