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Optimising pension financing with an application to the Finnish earnings-related pension scheme

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Finnish Centre for Pensions

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ABSTRACT

Demographic development with a deteriorating dependency ratio driven by increasing longevity and low fertility rates, are the main causes of expected increases in pension expenditure. Contribution rates are typically expected to increase to make current arrangements viable with an ageing population. Taking expenditures as given, there are two questions to be analysed: sustainable level and time path of the contribution rate. In a sustainable scheme the present value of expenditures and initial wealth should equal the present value of contributions. However, this leaves open the determination of the time pattern of contributions.

Financing of pensions is not only a one-to-one transfer of purchasing power from wage earners to pensioners but this involves direct collection costs and indirect misallocation costs induced by incentive effects on the working population. Financing of pensions thus involves the using up of resources that are referred to in public finance literature as dead weight loss or excess burden.

In this paper we analyse how to set optimality criteria for the contribution path taking expenditures as given. We derive results in the case of certainty and in a case where uncertainty about the future is approached in a simple way. We study how uncertainty about transitory or permanent shocks in the contributions base influences the setting of contribution rates. In characterising the implications of future uncertainty for setting the contribution rate, in the context of the Finnish pension scheme, it turned out that uncertainty with respect to relatively large permanent shocks have modest consequences for setting the level of contribution rate. Introducing relatively long-lived but temporary labour market shocks had only negligible impacts.

CONTENTS

1   Introduction

Demographic development with deteriorating dependency ratio driven by increasing longevity and low fertility rates, are the main causes of expected increases in pension expenditure. Contribution rates are typically expected to increase to make current arrangements viable with an ageing population. Expenditures on public pensions are projected to increase significantly as a proportion of GDP in most countries over the coming decades (OECD 2001).

Financing of pensions is not only a one-to-one transfer of purchasing power from wage earners to pensioners. The provision of benefits that are conditioned on income or employment and the collection of the pay-roll contributions needed to finance those benefits create deadweight losses that result from changing behaviour of the population (see Feldstein and Liebman 2002). Financing pensions thus involves direct collection costs and indirect misallocation costs induced by incentive effects on the working population.

In a sustainable scheme the present value of expenditures and initial wealth should equal the present value of contributions. However, this leaves open the determination of the time pattern of contributions. In this paper we analyse how to set optimality criteria for the contribution path taking expenditures as given. We derive results in the case of certainty and in a case where uncertainty about the future is approached in a simple way. We also study how uncertainty about transitory or permanent shocks in the contributions base influences the setting of contribution rates. It should be noted that we analyse the financing of a given scheme and do not discuss the optimality of the scheme as such.

We apply the results of our exercise to the Finnish statutory earnings-related pension scheme. In section four, after characterising an actuarial model of the Finnish pension scheme and its baseline projection, we introduce an optimal financing rule under certainty into the model and discuss the results briefly. After the certainty case we evaluate some results relating to uncertainty in the shocks to the employment rate. In section five we make some concluding remarks.

2 Optimising pension finances – setting the analytical framework

Financing of pensions is not only a one-to-one transfer of purchasing power from wage earn-ers to pensioners but this involves direct collection costs and indirect misallocation costs induced by incentive effects on the working population. Financing of pensions thus involves the using up of resources that are referred to in public finance literature as dead weight loss or excess burden. In this paper we analyse how to set a costs minimising path while taking expenditures as given.

2.1 Optimal contribution rate under certainty

We assume that the excess burden (B_i) at time i of financing pensions can be characterised by a function that has contributions revenue and the revenue base – that is wages – as its argu-ments:

$$B_i = \Phi\left(C_i, W_i\right) = W_i \Phi(\frac{C_i}{W_i}, 1) = W_i L\left(c_i\right) \tag{1}$$

The excess burden at period i is assumed to depend on current contributions (C_i) and wage sum (W_i) only. We also assume that the function, which characterises excess burden, is increasing in both of its arguments and is 1-order homogenous. This has a natural interpretation that if we have a constant contribution rate (C_i) doubling wages doubles the excess burden: the absolute size of costs grows proportionally to the size of the contribution base. $L(c_i)$ can be interpreted as unit loss function and the wage sum is the natural scaling factor for these losses. Unit losses are increasing with the contribution rate.

When optimising pension finances our problem is to find a sequence of pension contribution rates ($c_1, c_2...$) that minimises a convex loss function $L(c_i)$ subject to financing constraint.

$$\min \sum_{i=1}^{\infty} \beta^i W_i L(c_i)$$
$$\text{subject to} \quad A_0 + \sum_{i=1}^{\infty} \beta^i W_i c_i = \sum_{i=1}^{\infty} \beta^i W_i e_i \tag{2}$$

In problem (2) β is a discounting factor that is used to express variables in present value terms, e_i is pension expenses relative to wage sum, W_i is the wage sum, and A_0 the initial level of pension assets. The discounting factor is assumed to be the same in the objective function and budget constraint: the social discount rate in assumed to equal market rate of return. The constraint of the minimisation problem states that the present value of pension expenditures has to be equal with the present value of contributions and initial stock of pension wealth. The

discounted sum of losses is weighted by the annual wage sum. Thus, the minimised sum of losses depends on the size of the contribution base as well as on the contribution rate.

We have presented the problem with infinite horizon. This simplifies the treatment of terminal values in modelling exercises. We assume that in a relatively distant future the relative pension expenditures, e_i, is constant and the wage sum, W_i, is growing at a constant rate. If the discounting factor is higher than the wage growth the budget constraint as well as the objective function are converging to finite sums.

The optimisation problem can be solved using the Lagrange multiplier method. The first-order conditions of our problem implies (see Appendix) that

$$
L'\left(c_i^{opt}\right) = L'\left(c_j^{opt}\right) \quad i \neq j \quad \forall i, j
$$
$$
\Rightarrow c_i^{opt} = c_j^{opt}
$$

(3)

In the cost minimising solution the marginal cost for all periods is set equal. This implies that the contribution rate that minimises the loss function is the same for all periods. This result follows from the convexity of the objective function and it is analogous to Barro's (1979) tax-smoothing hypothesis in a pension finance context. The optimal contribution rate (c^{opt}) can be solved from budget constraint:

$$
c^{opt} = \frac{\sum_{i=1}^{\infty} \beta^i W_i e_i - A_0}{\sum_{i+1}^{\infty} \beta^i W_i}
$$

(4)

One implication of the above argument is that if there are expected permanent spending increases one should react immediately. Contribution rates should be set to the level that can be maintained at the time of higher expenditure level. Initially this implies accumulation of reserves and later on de-cumulation of them to finally find the level of reserves that remains constant relative to the wage sum. In an alternative scenario, negative assets might be an optimal outcome. If transitory expenditure rises are followed by a permanent decline, it could be optimal to accumulate a permanent debt to cover the short-lived expenditure peak.

One should note that the constant contribution result depends on the assumption of the equality of market and social discount rates. If social time preference is assumed to be lower than the market rate of return, future costs would have higher weight in the objective function and consequently current contribution rates should have higher value than future ones. In terms of the previous example this would imply faster accumulation of reserves and a lower steady state contribution rate finally relative to the constant rate case. Alternatively, if we prefer to put more weight on current costs, the initial accumulation of reserves should be slower and the ultimate steady state contribution rate higher than the current rate.

2.2   Optimal contribution rate under uncertainty

We evaluate uncertainty in a simplified way by assuming that up to a point T everything is known. At moment T we have a lottery that resolves alternative future outcomes. Each outcome defines an alternative path from date T to infinity. In this setup we minimise expected losses:

$$\sum_{i=1}^{\infty} E(\beta^i \underline{W}_i L(\underline{c}_i)) = \sum_{i=1}^{T} \beta^i W_i L(c_i) + \sum_{i=T+1}^{\infty} E(\beta^i \underline{W}_i L(\underline{c}_i)) \tag{5}$$

where E refers to expectation operator, and $\underline{W}_i$ and $\underline{c}_i$ to random variables of wage sum and contribution rate in an uncertain future.

We derive the solution to the above problem by backward induction. First we define the solution from date T to infinity when the lottery has taken place and uncertainty is resolved. After date T the problem is analogous to the one under certainty and contribution rates can be solved as in equation (4) shown in Appendix (A.5). In the second stage we solve the contribution rate for periods $[1,T]$ by the following formula:

$$L'(c_{T-}^{opt}) = E(L'(c_{T+}^{opt})) \tag{6}$$

Marginal losses for periods $[1,T]$ should equal the expected marginal losses for $[T,\infty]$. Details of the derivation of equation (6) are presented in appendix A. If the loss function is of the following form:

$$L(c) = \frac{1}{1+\gamma} c^{1+\gamma}, \qquad \gamma > 0 \tag{7}$$

it follows from Jensen's inequality that:

$$c_{T-}^{Opt} \begin{cases} < E(\underline{c}_{T+}^{Opt}) & when \, \gamma < 1 \\ = E(\underline{c}_{T+}^{Opt}) & when \, \gamma = 1 \\ > E(\underline{c}_{T+}^{Opt}) & when \, \gamma > 1 \end{cases} \tag{8}$$

With a quadratic loss function it is optimal to set the current contribution rate to equal the expected contribution rate. If the loss function is more convex the current rate should be higher that the expected rate and vice versa. The inevitable response to increased uncertainty is not to increase the current contribution rates. This depends on the shape of the objective function or the degree of risk aversion.

3 Applying optimal financing to the Finnish earnings-related pension scheme

3.1 Earnings-related pensions in Finland

The Finnish statutory pensions are made up of earnings-related pensions and national pensions, whereas voluntary pensions play a relatively minor role in the total pension provision. Earnings-related pensions strive to secure a reasonable income for the insured and their families in relation to the earnings while working in the event of old age, disability or death. The national pension guarantees a minimum income for pension recipients with no other pension income or with only a small earnings-related pension.

The earnings-related pensions are defined-benefit, in a sense that the size of the pension expenditure determines the contribution level and the need for other financing. The earnings-related pension scheme consists of several pension acts, which together cover the different sectors of the economy. The most important of these are the Employees Pensions Act, the State Employees' Pensions Act, the Local Government Pensions Act. The pension reform in 2005 to a large extent harmonised the determination of pensions under different pension acts. Different pension acts are needed because they each offer different ways to finance pensions.

An earnings-related pension accrues from earnings between the ages of 18 and 67. The insured is entitled to a normal old-age pension at the age of 63, but he or she can continue to work up to the age of 68 at an increased accrual rate. When calculating the initial amount of the pension, the earnings for the different years are adjusted in line with the wage coefficient, where the weighting of the change in the earnings level is 80 per cent and the weighting of the change in consumer prices is 20 per cent. Pensions are adjusted in line with an index where the weighting of the change in the earnings level is 20 per cent and the weighting of the change in prices is 80 per cent.

The initial amount of old-age pensions is adjusted to account for the change in longevity for 62-year-olds through the life expectancy coefficient. This life expectancy coefficient is determined so that the capital value of the old-age pension remains unchanged even if the life expectancy for persons at retirement age has changed in comparison to the expectancy calculated from the statistics for 2003–2007 as a reference value.

Financing pensions

Currently, the Finnish earnings-related pension scheme is a partially funded scheme which has about 110 billion euros worth of assets, that is, about 1.7 times the wage sum. About 70 per cent of these assets are owned by private-sector pension providers. Funds are invested both domestically and internationally in commercial assets. As opposed to many other schemes, the funds in the Finnish scheme are not individualised at employee level. The amount of funding is linked to each individual in order to determine the liability of each pension provider and the funds are used to pay the specific individual's pension once he or she retires. Yet it is not

individualised in the sense that an employee would be able to differentiate the funded part from the pay-as-you-go part or, even closer to "individual accounts" in the Anglo-Saxon sense.

Pension expenditure for self-employed persons and for farmers is financed through the annual premium income and the State's share. The State's share is the part of the pension expenditure which is not covered by the premium income, in practice this is covered by general tax collection. The magnitude of this component is shown in table 1.

Table 1. Statutory pension expenditures, contributions and assets in 2005 (%-share of wages).

	Wage sum (Million)	Pension expenditures	Pension contributions	State's share of expenditures	Miscellaneous items	All contributions	Pension assets
Private-sector employees	40,778	18.2%	21.0%	0.1%	0.9%	21.9%	174.9%
Self-employed	4,203	26.5%	17.4%	10.3%	0.0%	27.7%	1.0%
Public-sector employees	18,036	30.1%	26.7%	9.8%	0.6%	37.1%	161.9%
Total	63,017	22.2%	22.4%	3.5%	0.7%	26.6%	159.6%

The state and local government pension schemes were originally based on a pure pay-as-you-go system. The Local Government Pensions Institution started funding pensions in 1988 in order to curb the increase in pension contributions. The State Pension Fund was established in 1990 to prepare for the State's future pension expenditures. The aim of this Fund is to gather assets so that the cost burden caused by the pensions of the post-war baby-boomers can be lessened in the years when the pension expenditure is at its highest.

3.2 Baseline for expenditure projection in the Finnish pension scheme

The numerical applications below have been calculated using the long-term actuarial model of the Finnish Centre for Pensions (Biström et al. 2005). This model replicates the functioning of the earnings-related pension scheme. On the other hand, the descriptions of economic behaviour in the model are assumptions and not results.

The main assumptions relating to pensions calculation are:
   1) population forecast,
   2) employment forecast,
   3) growth in earnings level and
   4) yield on pension assets.

With the help of this model, it is possible to make calculations to meet the planning and forecasting needs of the Finnish earnings-related pension scheme. The most important results of the model are annual pension expenditure, contributions and funds for the calculation period.

The population in the baseline scenario follows the population forecast of Statistics Finland until 2050, which was published in autumn 2004. This forecast assumes that the total fertility rate is 1.8, net migration is 6,000 persons, which is about 0.12 per cent of the population in a year and that the current rate of decrease in mortality will continue in the future. The population forecast in our calculations has been continued from 2050 onwards as such, except that from 2050, the rate of decrease in mortality is expected to be halved.

Figure 1. Development of old-age dependency ratio (65+/[16-64]) in Europe and Finland.
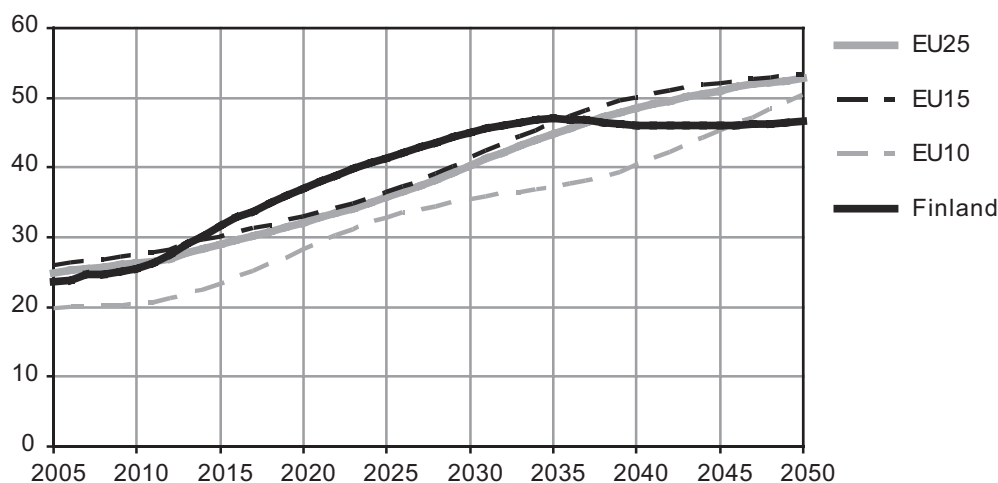
Source: Eurostat.

The current rate of decrease in mortality means a considerable increase in longevity in the long term. According to this forecast, by the year 2050, the life expectancy for men aged 62 years increases by almost 6 years and that for women by about four years and a half.

The development of the Finnish dependency ratio is compared to the European aggregate in figure 1. The old-age dependency ratio will double from the current level by 2030, and after that, the old-age dependency ratio will still increase a little. The Finnish population is ageing faster than the population on average in Europe. The diverging pattern is mainly explained by an early and relatively short post-war baby-boom. In the next few decades the old-age dependency ratio deteriorates earlier than in Europe on average but it will stabilise at a slightly lower level in the 2050s.

Assumptions of the baseline calculations are listed in table 2. Productivity growth (1.75%) and asset yields (4%) are assumed to be constant over the entire horizon. Macroeconomic assumptions regarding productivity growth and developments in participation rates are similar to those used by the OECD (2001) in a comparative study of fiscal implications of ageing.

Table 2. *Assumptions in the baseline scenario.*

Year	2005	2015	2025	2050
	Base year values	%-point changes from the base year 2005		
Employment rate	66.4	2.1	2.8	4.5
Unemployment rate	8.4	-2.7	-2.8	-2.9
Dependency ratio	24	9	18	24
Real wage/productivity growth	1.75	-	-	-
Yield of pensions assets	4.0	-	-	-

3.3   Optimal finances in baseline with certainty

In this section we characterise the baseline results under certainty. We take pension expenditures in the earnings-related pension scheme as given. They are determined by the current legislation, population dynamics and baseline development in the labour market. A significant factor determining the level of expenditures is the life expectancy coefficient that was introduced in the 2005 pension reform.

Figure 2 displays the development of expenditures and funds with optimal rate of contributions. All variables are expressed as a percentage share of the wage sum. The left-hand-side vertical axis is for contribution and expenditure shares and the right-hand-side axis for funds.

Figure 2. *Pension expenditures, contributions and assets in baseline.*
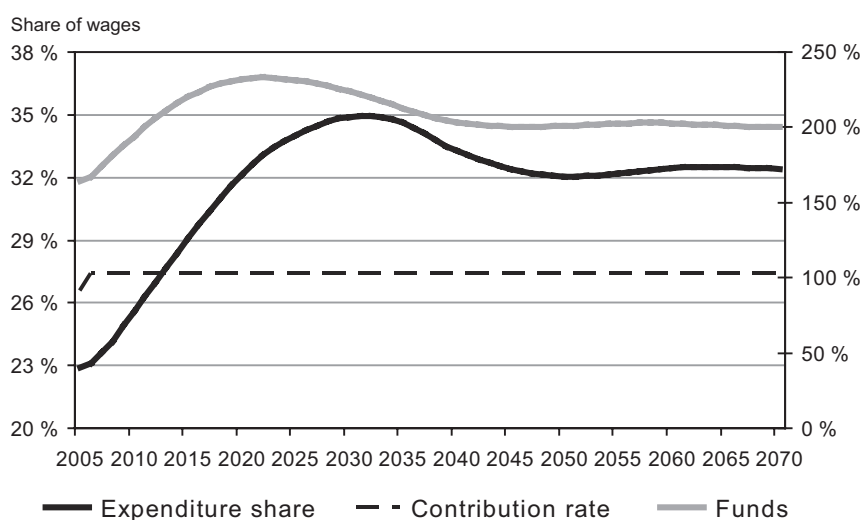
The expenditures are expected to become larger than the contributions in less than ten years. Relative to wages they are peaking around 2030, a few years earlier than the expected dependency ratio reaching its maximum in the population forecast. From the maximum share of 35 per cent expenditures are expected to gradually decline to a stable share around 32 per cent of wages.

Although expenditures exceed contributions after a short time, pension funds continue to grow a decade after this has taken place. The reason for this is that asset returns more than compensate for the gap between current expenditures and contributions. The stock of assets relative to wages is at the highest level close to 2020 and finds a stable share of 200 per cent around 2035. At the steady state phase, the funds contribution to annual pension financing is about 5 per cent relative to the wage sum.

The optimal contribution rate in the baseline under certainty is 27.4 per cent, which is very close to the total contribution rate in 2005, that is, 26.6 per cent of the wage sum (see table 1). In this respect the population challenge seems not be overwhelming for the partially funded Finnish earnings-related pension scheme.

*Table 3. Sensitivity of contribution rate to alternative assumptions on productivity and asset yield.*

	Asset yield		
Productivity growth	**3.5%**	**4.0%**	**4.5%**
1.50%	28.8%	27.7%	26.6%
1.75%	28.4%	27.4%	26.4%
2.00%	28.1%	27.1%	26.1%

In table 3 it is reported how sensitive the contribution rate is to the main assumptions on productivity growth and asset yield. The variability in productivity growth implies deviations from the mean value that are within the range from -0.4 to 0.4 percentage points in the contribution rate, whereas the variation in asset yield implies deviations within the range from -1.0 to 1.0 percentage points.

The current contribution rate is close to the optimal rate, and in some plausible scenarios even above it. However, one should immediately notice that a constant rate was not a policy imperative at the date the current scheme was founded. In the public sector the pension scheme was on a pure pay-as-you-go basis until the end of the 1980s. A pension scheme financed on a pure pay-as-you-go basis cannot fulfil the constant rate rule.

Figure 3 presents historical data on private-sector pension contributions and expenditures since the beginning of the system. The immediate impression from this figure is that even though there is a funding element in the scheme, the current contribution rate has mainly been adjusted to meet the needs to finance ongoing expenses.

*Figure 3. Private-sector pension expenditures as share of wages in 1962–2005.*
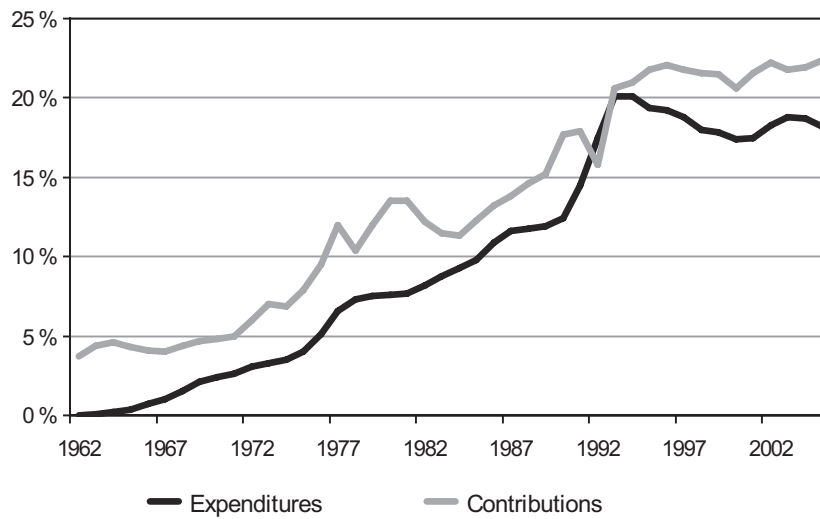
3.4 Optimal finances with uncertain future alternatives

In this section the behaviour of a pension scheme with optimal contribution rule is evaluated, if uncertainty about future development is taken into consideration when determining the contribution rate. Uncertainty is characterised in a very stylised manner in the following exercises. We assume that everything continues on a business-as-usual basis until 2015. From this point onwards we specify five alternative paths in employment and unemployment and attach discrete probabilities to each realisation. The same logic could have been used to analyse uncertainty in interest rates or wage development.

The alternative outcomes relative to the baseline are temporary boom, temporary slowdown, permanent expansion and permanent stagnation. Alternative paths are characterised in figure 4, where four alternative employment paths are displayed relative to the baseline.

Figure 4. Alternative employment paths characterising uncertainty.
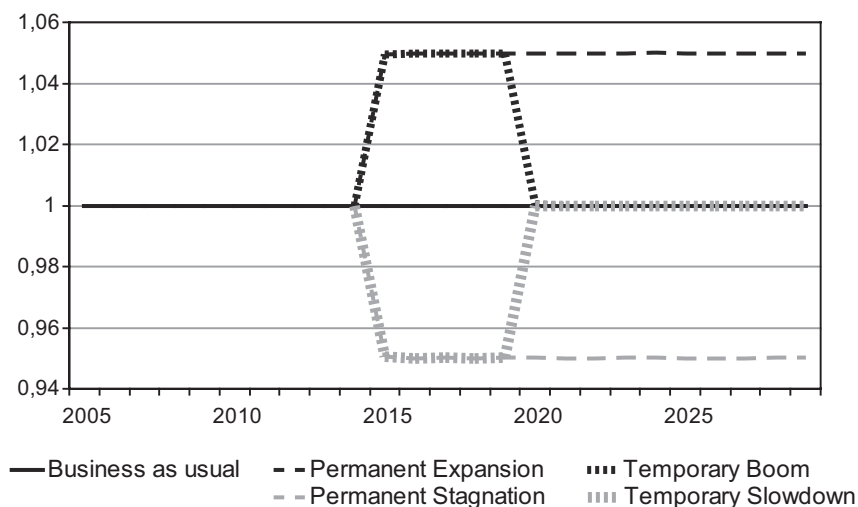
In the temporary boom we assume that the employment rate increases by 5 per cent of which 60 per cent comes from decreased unemployment and 40 per cent from increased participation in the labour market. This is assumed to take place for the period 2015–2020. After five years, employment returns to the baseline. Slowdown is characterised by similar figures in the opposite direction. The size of permanent shocks are assumed to be same as that of temporary ones but they are added as perpetual deviations to the baseline from 2015 on. An important assumption that has to be kept in mind, when interpreting the results, is that labour market status – whether employed or unemployed – has no impact on retirement.

We have calculated the pre- and post-shock contribution rates using different beliefs concerning the likelihood of future events. The subjective probability distributions concerning the alternative future outcomes are displayed in table 4. In the first alternative we have assumed symmetric distributions of alternative future paths. In the last two cases we have assumed asymmetric distributions for the expected outcomes. The first case puts more weight on positive events whereas the latter has more gloomy expectations about future prospects.

Table 4. *Alternative probability distributions on beliefs of uncertain outcomes.*

	Permanent Stagnation	Temporary Slowdown	Baseline	Temporary Boom	Permanent Expansion
(1) Symmetric	0.1	0.25	0.3	0.25	0.1
(2) Optimistic	0.1	0.1	0.3	0.25	0.25
(3) Pessimistic	0.25	0.25	0.3	0.1	0.1

Table 5 reports optimal contribution rates before and after the realisation of alternative paths. We have calculated the pre- and post-shock contribution rates using formula (A.8) in Appendix. This formula has been applied to different beliefs concerning the likelihood of future events. In deriving these rates we have assumed a quadratic loss function of the form $L = 0.5 \cdot c^2$. Applying quadratic loss function implies that contribution rates for the period 2006–2015 are the expected values of alternatives after 2015.

In the first two alternatives we have assumed symmetric distributions of alternative future paths. Although the weights of future events vary the setting of contribution rates are identical to the second decimal of a percentage point. In the last two cases we have assumed asymmetric distributions for the expected outcomes. The first case puts more weight on positive events whereas the latter has more gloomy expectations about future possibilities.

**Table 5.** *Optimal contribution rates for alternative events.*

	2006–2015	2015–				
		Permanent Stagnation	Temporary Slowdown	Baseline	Temporary Boom	Permanent Expansion
(1) Symmetric expectations	27.40%	27.87%	27.45%	27.39%	27.33%	26.98%
(2) Optimistic expectations	27.35%	27.90%	27.47%	27.41%	27.35%	27.00%
(3) Pessimistic expectations	27.46%	27.86%	27.43%	27.38%	27.32%	26.97%

In the case of temporary shocks the rates are almost identical to the baseline. As a matter of fact, in the temporary slowdown there is no discrepancy at the accuracy of one decimal point. The case of permanent stagnation necessitates 0.5 per cent upward adjustment in the contribution rate and that of permanent expansion allows adjusting the contribution rate downward by 0.4 per cent. In the results reported below we have used the contribution rates from alternative (1) in table 5.

Temporary shocks

Temporary shocks on employment and unemployment, although taking place for a substantial time period, do not have any significant lasting effects on pension expenditures as a share of wages. The impact of these shocks on pension expenditures is presented in figure 5. In the case of slowdown, when unemployment increases and people are withdrawing from the labour market, the expenditure share initially increases. The immediate impact of labour market development on pension expenditures is very limited. On the other hand, the wage sum shrinks significantly because of increased unemployment and decreased labour market participation. The reverse is true with the temporary boom.

Figure 5. *Pension expenditure shares in the case of temporary shocks.*
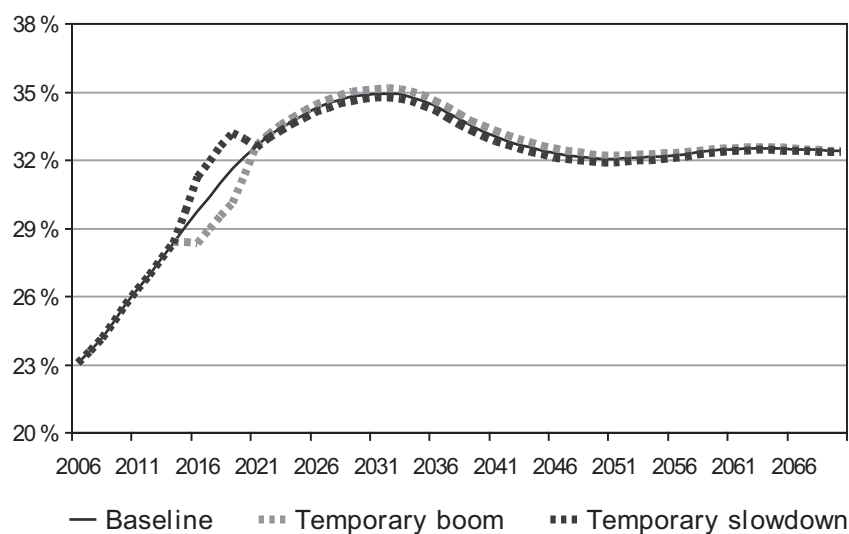
In the longer term, lost income because of labour market slack diminishes pension expenditure, because during the slack total wage earnings and consequently accrual of pension rights are smaller. This is reflected in the future as a slightly smaller share of pensions in the temporary slowdown alternative relative to the baseline and the mirror image effect in the case of boom.

Figure 6 displays the development of pension fund assets for the baseline and temporary shocks. Everything that was said about expenditures applies also here. The immediate changes in the asset stock are relatively minor compared to changes in the wage sum. Temporary but fairly long-lasting labour market turbulence has lengthy transition implications for a stable level of assets.
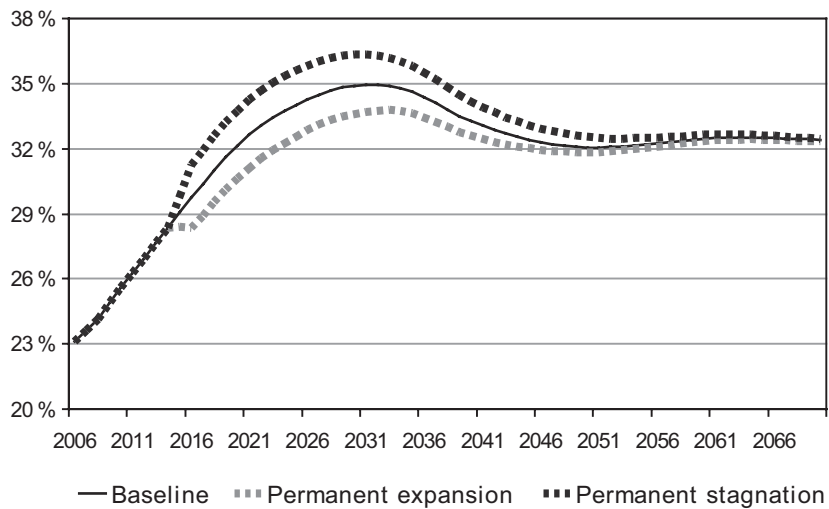
*Figure 6. Funds as %-shares of wage sum in the case of temporary shocks.*
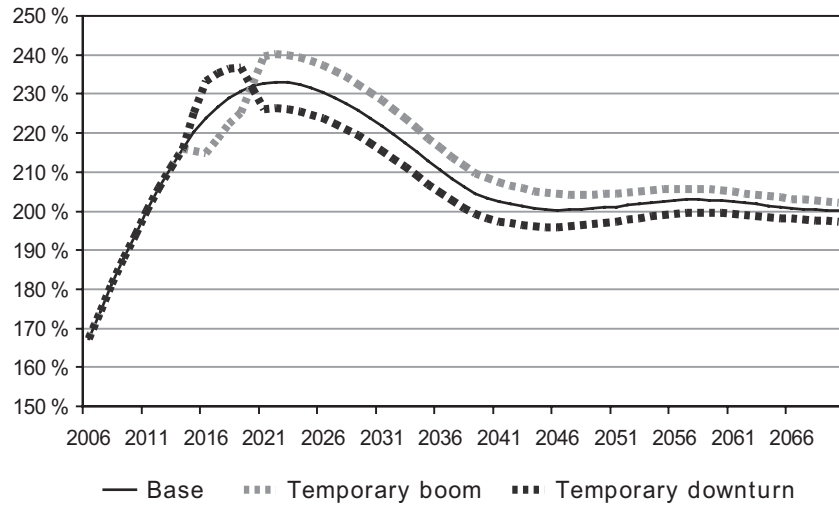
Permanent shocks

Patterns of pension expenditures when labour market changes are permanent are displayed in figure 7. It turns out that if measured relative to wages the impact of shocks are not persistent. Benefits in this scheme depend linearly on earned income. The reason for a temporarily lower wage share of benefits in the permanent expansion case depends on the time that is needed for complete exit of those pensioners who have not been affected at all or benefited only partially from the labour market shock under consideration. This takes about 40 years, which is approximately one generation, to pass through to the stable expenditure level.

Figure 7. Pension expenditure shares in the case of permanent shocks.

Figure 8. *Funds as %-shares of wage sum in the case of permanent shocks.*
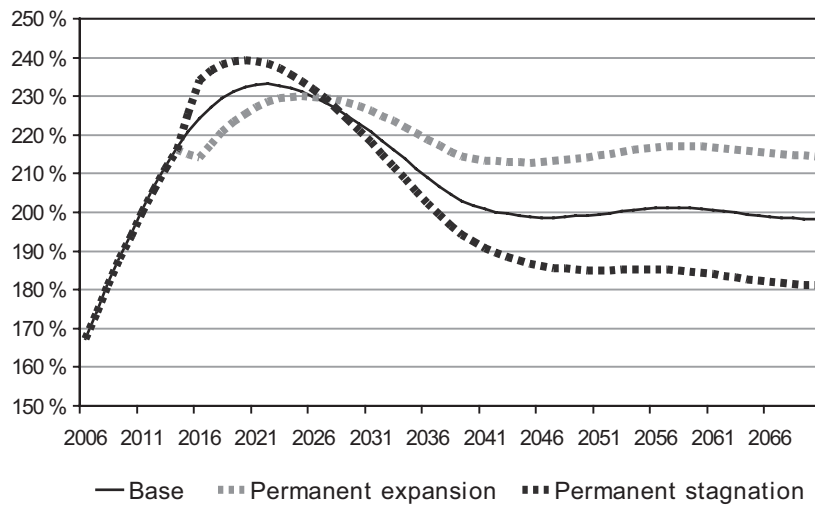
4   Concluding remarks

In this paper optimal pension financing under demographic challenges and economic uncertainty has been analysed from one particular point of view. We have asked what kind of policy would minimise allocation distortions in financing pension expenditures that are on a defined benefit basis. It turned out that a constant contribution rate over the planning horizon will minimise these costs. In the case under uncertainty we have a corresponding result with quadratic loss function: the optimal rate is equal to the expected rate.

However, the strong result on the constant rate assumes that the market and social discount rates are equal.

Numerical applications with the Finnish earnings-related pension scheme were carried out. In characterising the implications of future uncertainty for setting the contribution rate it turned out that uncertainty with respect to relatively large permanent shocks have modest consequences for setting the level of contribution rate. Introducing relatively long-lived but temporary labour market shocks had only negligible impacts.

The financing implications depend on the earnings-related nature of the scheme: benefit accruals are linearly dependent on earned income. The impact of shocks have no or small persistent implications for contribution rates. Pension assets, which are a component of the scheme, are acting as a buffer to absorb the shocks under study. Sensitivity analysis with asset yield values was also performed. It turned out that the variability of asset yields is a more important determinant for the contribution rate than the growth rate of labour productivity, if one considers the range of values that is typically used in these types of exercises.

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Appendix

Optimal contributions under certainty

The objective is to find a sequence of pension contribution rates $c_1,...,c_N$ that minimises a convex loss function $L(c_i)$ subject to financing constraint.

$$\min \sum_{i=1}^{\infty} \beta^i W_i L(c_i)$$

$$\text{subject to} \quad A_0 + \sum_{i=1}^{\infty} \beta^i W_i c_i = \sum_{i=1}^{\infty} \beta^i W_i e_i \tag{A.1}$$

In problem (2) β^i is a discounting factor that is used to express infinite sums in present value terms, e_i is pension expenses relative to wage sum, W_i is the wage sum, and A_0 the initial level of pension funds. The Lagrangean function of the constrained optimisation problem is:

$$\Lambda = \sum_{i=1}^{\infty} \beta^i W_i L(c_i) - \lambda \left(A_0 + \sum_{i=1}^{\infty} \beta^i W_i c_i - \sum_{i=1}^{\infty} \beta^i W_i e_i \right) \tag{A.2}$$

The first-order conditions of the problem are:

$$\frac{\partial \Lambda}{\partial c_i} = \beta^i W_i \frac{\partial L(c_i)}{\partial c_i} - \lambda \beta^i W_i = 0 \Rightarrow L'(c_i) = L'(c_j) \tag{A.3i}$$

$$\frac{\partial \Lambda}{\partial \lambda} = A_0 + \sum_{i=1}^{\infty} \beta^i W_i c_i - \sum_{i=1}^{\infty} \beta^i W_i e_i = 0 \Rightarrow c = \frac{\sum_{i=1}^{\infty} \beta^i W_i e_i - A_0}{\sum_{i+1}^{\infty} \beta^i W_i} \tag{A.3ii}$$

If the growth rate of wages is smaller than the interest rate the infinite sum converges to a finite number and contribution rate (c) can be solved from 3(ii).

Optimal contributions under uncertainty

The objective is to find a minimum for the following objective function

$$\sum_{i=1}^{\infty} E(\beta^i \underline{W}_i L(\underline{c}_i)) = \sum_{i=1}^{T} \beta^i W_i L(c_i)) + \sum_{i=T+1}^{\infty} E(\beta^i \underline{W}_i L(\underline{c}_i)) \tag{A.4}$$

where everything is known up to point T. We have uncertainty about the future development path after that date. We have a known distribution of alternative development paths which are resolved in a lottery at date T. After this lottery has taken place everything is known for sure from that date to infinity.

Since everything is known after time T the problem is analogous to the problem with certainty and we know the answer for that:

$$c_{T+}^{opt} = \frac{\sum_{i=T+1}^{\infty} \beta^i W_i e_i - A_T}{\sum_{i=T+1}^{\infty} \beta^i W_i} \qquad (A.5)$$

We can derive the asset level at T from the budget constraint:

$$A_T = A_0 + c_{T-} \sum_{i=1}^{T} \beta^i W_i - \sum_{i=1}^{T} \beta^i W_i e_i \qquad (A.6)$$

where the contribution rate for the first T periods (c_{T-}) is unknown. When (A.5) and (A.6) is inserted in (A.4) we get

$$\sum_{i=1}^{T} \beta^i W_i L(c_i)) + \sum_{i=T+1}^{\infty} E(\beta^i \underline{W}_i L(\underline{c}_i)) =$$

$$\sum_{i=1}^{T} \beta^i W_i L(c_{T-})) + \sum_{i=T+1}^{\infty} E \beta^i \underline{W}_i L \left(\frac{\sum_{i=T+1}^{\infty} \beta^i \underline{W}_i \underline{e}_i - A_0 - c_{T-} \sum_{i=1}^{T} \beta^i W_i + \sum_{i=1}^{T} \beta^i W_i e_i}{\sum_{i=T+1}^{\infty} \beta^i \underline{W}_i} \right) \qquad (A.7)$$

From this function the first-order conditions for the optimum is

$$L'(c_{T-}^{opt}) = E(L'(\frac{\sum_{i=T+1}^{\infty} \beta^i \underline{W}_i \underline{e}_i - A_0 - c_{T-}^{opt} \sum_{i=1}^{T} \beta^i W_i + \sum_{i=1}^{T} \beta^i W_i e_i}{\sum_{i=T+1}^{\infty} \beta^i \underline{W}_i})) \qquad (A.8)$$

and using (A.5) and (A.6) turns out to be:

$$L'(c_{T-}^{opt}) = E(L'(c_{T+}^{opt})) \qquad (A.9)$$